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Development of Special Functions in Mathematical Physics

Bhagvat K. Kumthekar[1], Vijay B. Patare[2]

Department of Physics, Nutan Mahavidyalaya, Sailu Dist Parbhani, India[1]

Department of Mathematics, Nutan Mahavidyalaya, Sailu Dist Parbhani, India[2]

ABSTRACT: In the present paper, I would like to explain special functions and their developments in mathematical physics. Such asBessel equation, Legendre equation, Laguerre equationand Hermite equation. These functions are developed only for special cases of equations. So that the names where given special functions.

I. INTRODUCTION

There are some special functions in mathematics which are developed due to requirement of special cases in Physics. In science and technology there are large number problems which are in terms of differential equations or partial differential equations. Partial differential equations have two ormore independent variables. When a function f is depending upon number of variables (x,y,z,t)then differentiation with respect to one of the variable is known as partial differentiation e.g. $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial t}$. If our function is depend upon only one variable and we differentiate with respect to that variable it becomes perfect differentiation$\frac{df}{dx}$.

A powerful tool for solving a partial differential equation may be to split it into (ordinary) differentialequations by the method of the separation of variables.

A relation between independent variable, say x, dependent variable, sayy, and derivatives of y with respect to x is known as a differential equation.

$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^ny}{dx^n}\right) = 0 \qquad (1)$$

Order of the given differential equation is the order of the highestderivative in the equation.Degree of the given differential equation is the degree (power) ofthe highest derivative in the equation [1].Differential equations may be classified into two categories:

(i) Linear differential equations, and
(ii) Non-linear differential equations.

$$a_n \frac{d^ny}{dx^n} + a_{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1 \frac{dy}{dx} + a_0 y = f(x) \qquad (2)$$

where a's may be functions of x or constant, but not the function of y. Here, each term contains either y or derivative of y with respect to x, and there is only one degree of y and its derivatives[2]. When $f(x)$ is zero, in above equation (2) is known as homogeneous linear differential equation. Differential equations, other than that of the form of

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equation (2) are nonlinear ones. Thus, in a non-linear differential equation, at least one term contains either product of y and its derivative, product of two or more derivatives, or has y or its derivative of degree more than one.

If $y_1, y_2, \ldots, y_r$ are r solutions of a homogeneous linear differential equationthen a linear combination of the solutions also is a solution of the equation[3].In the present communication I would like to discuss about Bessel equation, Legendre equation, Laguerre equation and Hermite equation.

1. **Legendre Equation:**

Legendre differential equation is,

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \qquad (3)$$

Where n is positive integer. Solution of this equation is known as Legendre Function[4]. In order to find out singular points and possibility of the series solution for this equation, we have

$$P(x) = -\frac{2x}{1-x^2} \quad \text{and} \quad Q(x) = \frac{n(n-1)}{1-x^2}$$

Such type of equation comes in Schrodinger's equation for Hydrogen atom in spherical polar coordinate [5].

Applications of Legendre functions

Legendre's equation occurs in many areas of applied mathematics, physics and chemistry in physical situation with a spherical geometry such as flow of an ideal fluid past a sphere, the determination of the electric field due to a charged sphere and the determination of the temperature distribution in a sphere given its influence. Legendre polynomials have an extensive usage area, particularly in physics and engineering. For example, Legendre and Associate Legendre polynomials are widely used in the determination of wave functions of electrons in the orbits of an atom, [5] and in the determination of potential functions in the spherically symmetric geometryetc. Also in nuclear reactor physics, Legendre polynomials have an extraordinary importance [6].

2. **Laguerre equation:**

Laguerre's differential equation is

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0\ldots \qquad (4)$$

Where n is positive integer. Solution of this equation is known as Laguerre Function [4]. In order to find out singular points and possibility of the series solution for this equation, we have

$$P(x) = \frac{1-x}{x} \quad \text{and} \quad Q(x) = \frac{n}{x}$$

Such type of equation comes in Schrodinger's equation for Hydrogen atom in spherical polar coordinate.

Applications of Laguerre's functions

A spectral method of inverting one- and two-dimensional semi-infinite convolutions using the Laguerre polynomials.Applying the Laguerre polynomials makes it possible to eliminate the discretization procedure, which may take the solution of the perturbed equation outside the region of well-posedness [7].Tempered fractional diffusion equations (TFDEs) involving tempered fractional derivatives on the whole space [8].

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3. **Hermite equation:**

Hermit's differential equation is

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0 \qquad (5)$$

Where n is positive integer. Solution of this equation is known as Legendre Function [4]. In order to find out singular points and possibility of the series solution for this equation, we have

$$P(x) = -2x \qquad \text{and} \quad Q(x) = 2n$$

Applications of Hermite's functions

Harmonic oscillator problem in quantum mechanics can be solved by using Hermite polynomials. The problem is usual: to find all values of energies$E$ for which such a non-trivial solution exists. The physical meaning of eigenvalues E_k is the energy levels, and eigenfunctions y are the "wave functions" which describe the state of the system [9]. The Landau levels, appearingin the quantum analysis of an electron (or any charged particle) moving in aclassical magnetic field. Morse potential used to treat the dynamicalbehavior of diatomic molecules and its approximation in terms of harmonicoscillator potential. [10]

4. **Bessel functions:**

Bessel's differential equation is

$$x^2\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + (x^2 - n^2)y = 0$$

where n is an integer or a half integer. Solution of this equation is known as Bessel function [4].In order to find out singular points and possibility of the series solution for this equation, we have

$$P(x) = \frac{1}{x} \qquad \text{and} \quad Q(x) = 1 - \frac{n^2}{x^2}$$

Applications of Bessel functions

Bessel's equation arises when finding separable solutions to Laplace's equation and the Helmholtz equation in cylindrical or spherical coordinates. Bessel functions are therefore especially important for many problems of wave propagation and static potentials. In solving problems in cylindrical coordinate systems, one obtains Bessel functions of integer order ($\alpha = n$); in spherical problems, one obtains half-integer orders ($\alpha = n + 1/2$). [11]For example:

- Electromagnetic waves in a cylindrical waveguide
- Heat conduction in a cylindrical object
- Diffusion problems on a lattice
- Solutions to the radial Schrödinger equation (in spherical and cylindrical coordinates) for a free particle

II. DISCUSSION

In many areas of applied mathematics, physics and chemistry in physical situation with a spherical geometry, determination of wave functions of electrons in the orbits of an atom. These functions are developed only for special cases of equations. So that the names where given special functions. It is easy to eliminate the discretization procedure, which may take the solution of the perturbed equation outside the region of well-posedness by using Laguerre polynomials. Bessel function is used for finding the solution of radial part Schrodinger equation for hydrogen atom. Associated Legendre function used for solving azimuthal part of Schrodinger equation for hydrogen atom.

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VARIATION OF DAMPING LENGTH AND WAVELENGTH IN NORTH POLAR CORONAL HOLES OF SOLAR ATMOSPHERE

Bhagvat K. Kumthekar
Department of Physics, NutanMahavidyalaya, SailuDistParbhani -431 503 (MS) India
Email: bkumthekar@gmail.com

ABSTRACT :

In the present paper, I would like to discuss about variation of damping length and wavelength in the region of 1.05R⊙ to 1.35R⊙ and in different cases like (i) Magnetic diffusivity only (ii) viscosity only and (iii) when both are present. It shows that Comparison of the fast and slow modes explicitly shows that the damping length as well as the wavelength for the fast-mode waves is much larger than those for the slow-mode waves.

INTRODUCTION:

The role of magnetohydrodynamics (MHD) waves has been discussed extensively in solar physics for understanding the outstanding problems of solar coronal heating and the solar wind accelerationMechanisms[1]. For derivation of dispersion relation we have to consider following MHD equation

$$\rho\frac{\partial v}{\partial t}+\rho(v\cdot\nabla)v=\frac{1}{\mu}(\nabla\times B)\times B+\rho\upsilon\nabla^2 V \qquad (1)$$

$$\frac{\partial B}{\partial t}=\nabla\times(v\times B)+\eta\nabla^2 B \qquad (2)$$

$$\nabla\cdot B=0 \qquad (3)$$

The equations (1), (2) and (3) are known as momentum equation, Induction equation and Magnetic flux conservation equation respectively. Where v is the velocity, B the magnetic field and ρ, μ,η, V are, respectively, the mass density, Magnetic permeability, magnetic diffusivity and the coefficient of viscosity.

Taking the perturbations from the equilibrium (Priest [2]) and linearize the equations (1) through (3) by neglecting squares and products of the small quantities. After solving above equations we get a dispersion relation as follows

$$\omega^2=k^2\left[v_A^2-i\omega(\upsilon+\eta)\right]+\upsilon\eta k^4 \qquad (4)$$

Where $V_A = B_0/(sqrt\{\mu\rho_0\})$ is the Alfven velocity. The dispersion relation was obtained by Pekunu et al. [3] &Kumthekar [4] . This dispersion relation is applied for the plasma in the North Polar Coronal Hole where assumed the angular frequency ω to be a real quantity and the wave number *k* as a complex quantity.

RESULT AND DISCUSSION:

For a given value of ω = 2π/τ and the physical parameters discussed in the Kumthekar [3] and solved with the help of a FORTRAN program. I assumed the angular frequency ω to be a real quantity and thewave

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number k as a complex quantity so that $k = kr + iki$. When both the $kr$and $ki$are positive numbers, they are related to the damping length D and the wavelengthλ of the wave as Chandra et al. [5]

$$D = \frac{2\pi}{k_i} \quad and \quad \lambda = \frac{2\pi}{k_r}$$

Equation (4) is a simple quadratic equation it can be solved for getting roots of equation. This equation can be studied for three cases (i) Magnetic diffusivity only (ii) viscosity only and (iii) when both are present.

Case I: Magnetic diffusivity:

Let us consider the case of magnetic diffusivity only. That is, there is no viscosity ($\upsilon = 0$). For this case, equation (4) gives $kr$and kivalues. Then we can apply a condition $\omega \ll v_A^2$, we get

$$k_r = \frac{\omega}{v_A} \quad and \quad k_i = \frac{\omega^2}{2v_A^3}$$

Thus, we have two roots; one with positive values of kr and ki, and the other with the negative values. For the positive values, the damping length D and wavelength λ are calculate as a function of R, for $\tau = 10^{-2}$ s, 10^{-3} s and 10^{-4} s and are given in Fig.1 (a). As from the expressions, here the damping length is much larger than the wavelength. While D shows a large variation, with a maximum around 1.2R⊙, the λ remains nearly constant. Fig.1 (a) shows that the wavelength is proportional to τ whereas the damping length is proportionalto τ^2.

Case II: Viscosity only:

Let us consider the case of viscosity only. That is, there is no magnetic diffusivity ($\eta = 0$). For this case, equation (4) gives $kr$and kivalues. Then we can apply a condition $\omega\upsilon \gg v_A^2$, we get

$$k_r = \frac{\sqrt{\omega}}{\sqrt{2\upsilon}} and k_i = \frac{\sqrt{\omega}}{\sqrt{2\upsilon}}$$

Again, we have two roots; one with positive values of kr and ki, and the other with the negative values. For the positive values, the damping length D and wavelength λ are calculate as a function of R, for $\tau = 10^{-2}$ s, 10^{-3} s and 10^{-4} s and are given in Fig.1 (b).

As from the expressions, as long as $\omega\upsilon \gg v_A^2$, the damping length and the wavelength are equal to each other. There is a maximum around $1.2R$⊙. It is the case for $\tau = 10^{-3}$ s and 10^{-4} s. But, for $\tau = 10^{-2}$ s, the D and λ differ slightly from each other, showing that the condition $\omega\upsilon \gg v_A^2$ is not satisfied here. There is a maximum around $1.2R$⊙.

Case III: Both are Present $\upsilon \neq 0$ and $\eta \neq 0$.

Equation (4) is a quadratic equation in k2 and therefore, its roots are of the form of two pairs: ±(kr1 +iki1) and ±(kr2 +iki2). These two pairs for the roots correspond to the fast-mode and slow-mode waves. The damping length D and wavelength λ for the two modes as a function of R for $\tau = 10^{-2}$ s, 10^{-3} s and 10^{-4} s are shown in Fig. 2. It is interesting to note that the variationof D and λ for the fast-mode wave is similar to that for the case of $\eta = 0$. Theslow-mode wave shows an opposite behaviour for the variation of D as well as λ. There is a minimum around $1.2R$⊙.Here, also, D is nearly equal to $\lambda$for $\tau = 10^{-3}$ s and 10^{-4} s. But, for $\tau = 10^{-2}$s , the D and λ differ slightly fromeach other. Comparison of the fast and slow modes explicitly shows that the damping length as well as the wavelength for the fast-mode waves is much larger than those for the slow-mode wave. Thus, the slow-mode waves cannot propagate through the corona.

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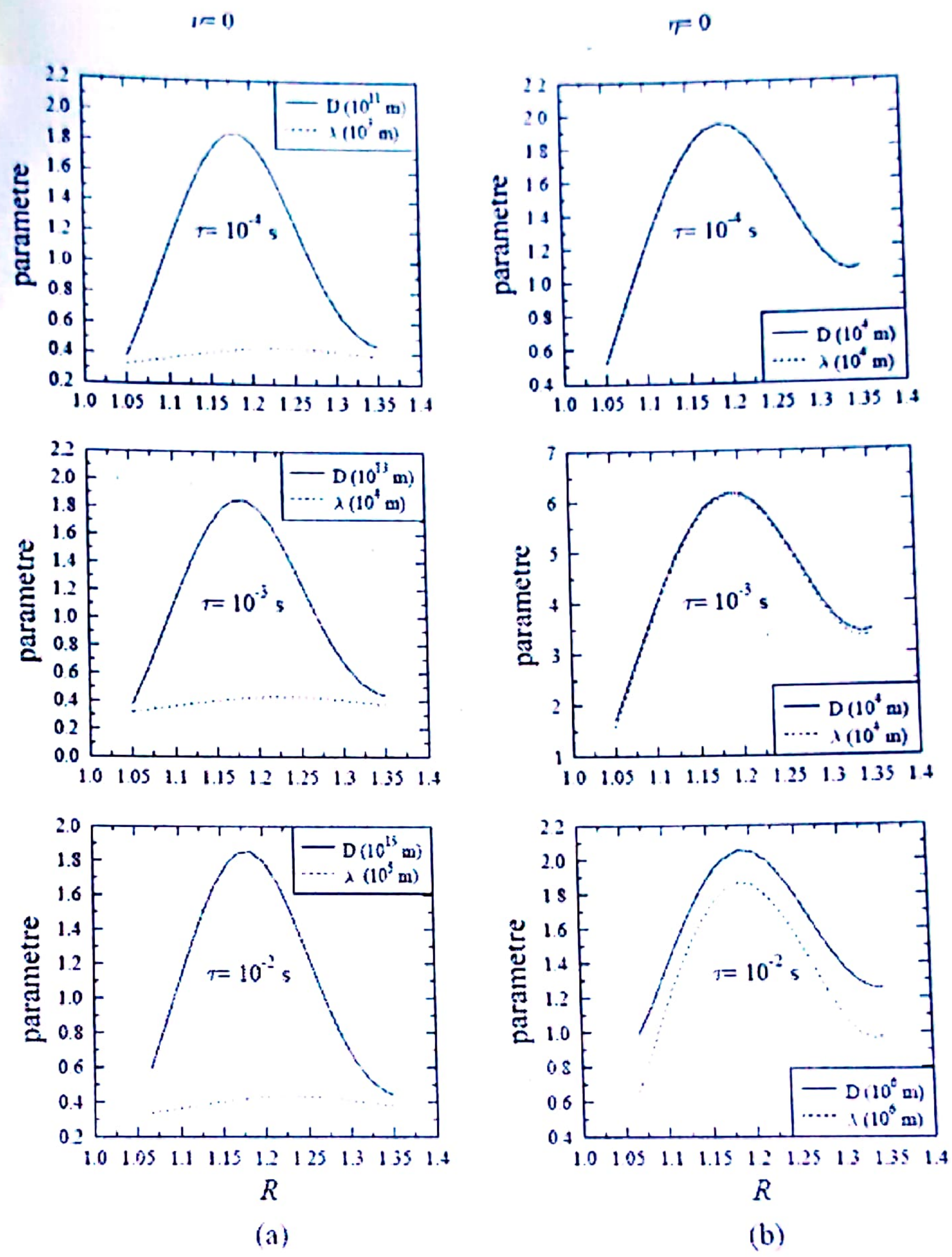

Fig. 1:Variation of damping length *D* and the wavelength λ as function of R for $\tau = 10^{-2}$ s, 10^{-3} s and 10^{-4} s for two cases υ=0 and η =0.

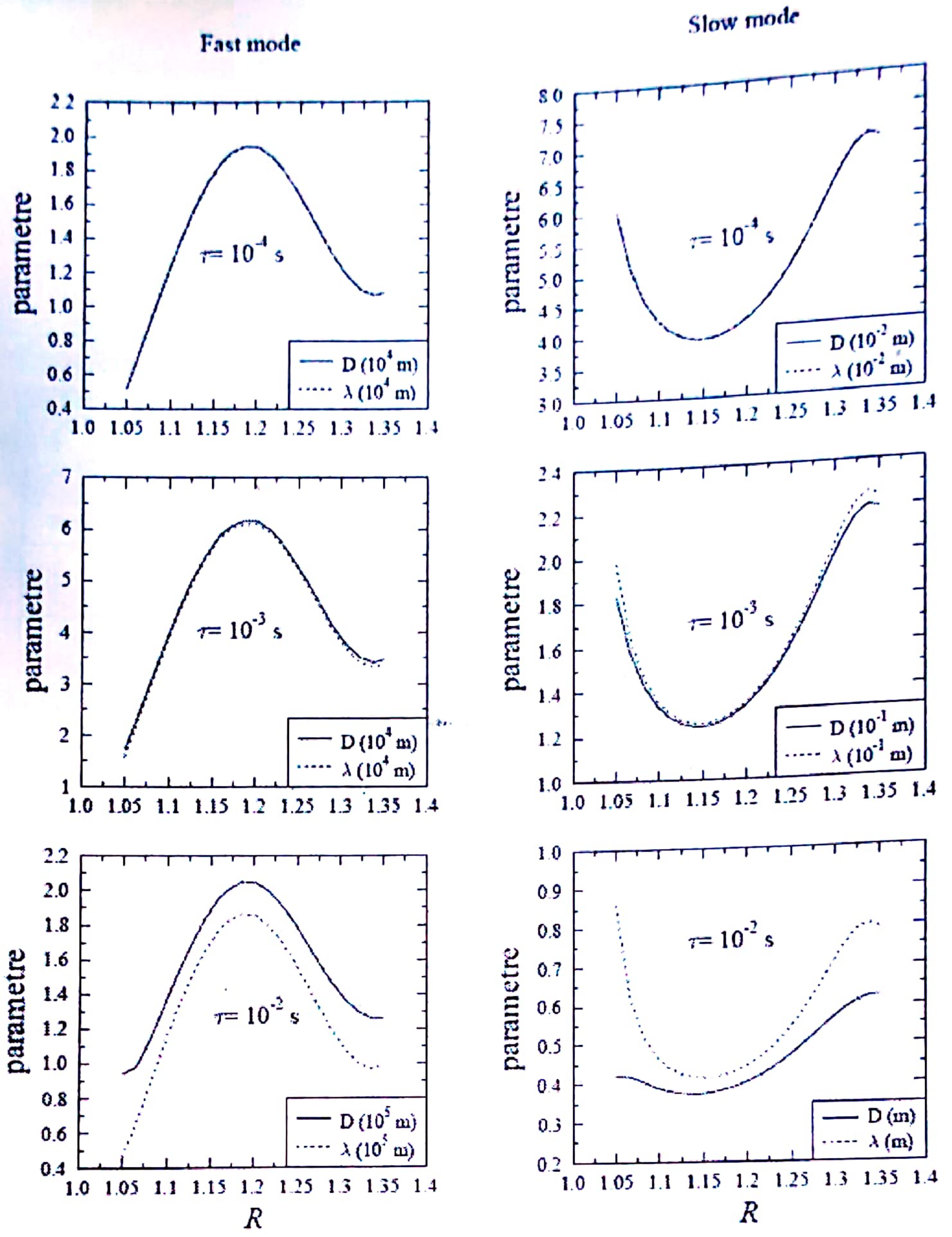

Fig. 2:Variation of damping length *D* and the wavelength λ for fast mode and slow mode waves as function of R for $\tau = 10^{-2}$ s, 10^{-3} s and 10^{-4} s for two cases $\upsilon \neq 0$ and $\eta \neq 0$.

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